Prediction and Tolerance Intervals

(Section 5.3 of Vardeman and Jobe)

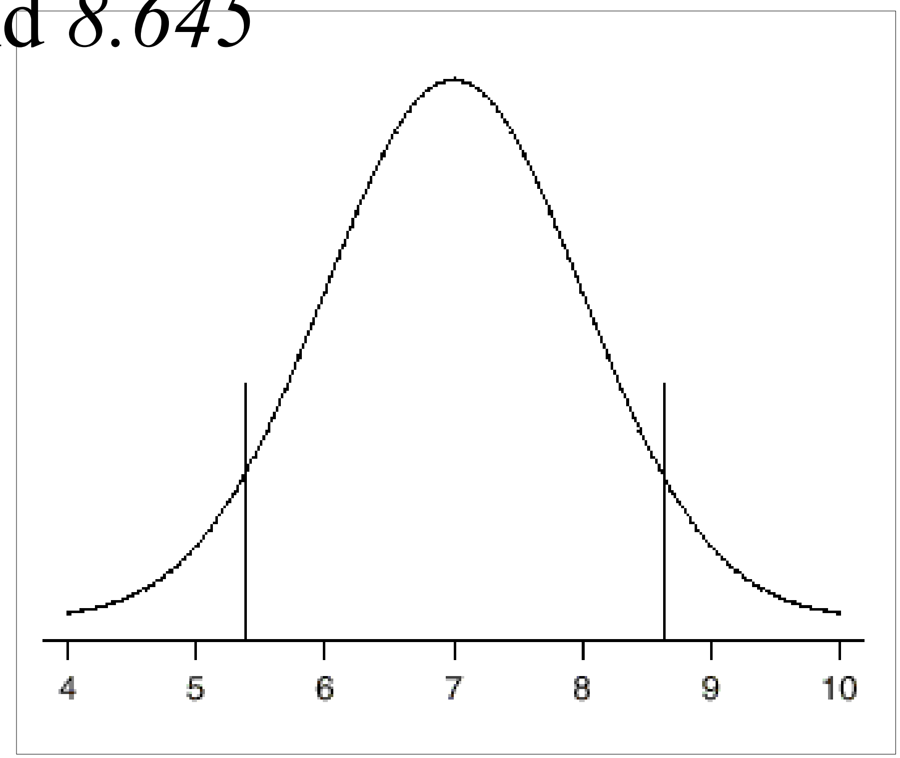
Two Other Methods of Characterizing a Process

- Prediction of where an additional value generated by the process will fall
- Location of “most” of the process output distribution

For a process with a “known” distribution these are essentially the same problem

$m = 7$ and $s = 1$ Normal Example

- There's a "90% chance" the next *x* is between *5.355* and *8.645*
- 90% of the process distribution is between *5.355* and *8.645*



What if I'm Not Omniscient?

- When one has to use a sample to get an approximate picture of a process, it is important to hedge such statements in light of sample variability/uncertainty … this can be done
  - for normal processes using $\bar{x}$ and s
  - in general using the sample minimum and/or maximum values

Limits for Normal Processes

- "Prediction limits" for a single additional *x* are

$$\bar{x} \pm ts\sqrt{1 + \frac{1}{n}}$$

- "Tolerance limits" (two-sided) for a specified fraction of the process output are

$$\bar{x} \pm t_2 s$$

for t_2 from Table A.9.a (one-sided limits are similar, but using Table A.9.b)

Example 5.6

- Angles of *n=50* holes drilled by EDM had $\bar{x} = 44.117$ and $s = .983$
  - Using *n-1=49* degrees of freedom (and a statistical package) to find the upper .025 point of the *t* distribution, 95% two-sided prediction limits for the next angle are

$$44.117 \pm 2.0096(.983)\sqrt{1+\frac{1}{50}}$$

i.e. 42.12 and 46.11

  - Using Table A.9.a, 99% two-sided tolerance limits for 95% of all angles are

$$44.117 \pm 2.580(.983)$$

i.e. 41.58 and 46.65

Limits Based on Sample Minimum and Maximum Values

- The interval $(\min x_i, \max x_i)$ can be used
  – as a prediction interval for a single additional *x*, and the associated prediction confidence level is then

$$\frac{n-1}{n+1}$$

  – as a tolerance interval for a fraction *p* of the process output, and the associated confidence level is then

$$1 - p^n - n(1-p)p^{n-1}$$

Example 5.6 continued

- The $n=50$ angles in Table 5.7 have minimum value *42.017* and maximum value *46.050*, so the interval *(42.017,46.050)* has
  - associated confidence level

$$\frac{50-1}{50+1}=96.1\%$$

as a prediction interval for the next angle (normal distribution or not)

Example 5.6 continued

– associated confidence level

$$1-(.95)^{50}-50(.05)(.95)^{50-1}=72.06\%$$

as a tolerance interval for, say, 95% of all angles

(normal distribution or not)

Workshop Exercise

- Do problems 5.17a), 5.17b), 5.18a) and 5.18b) for the nominally 3-inch saddles

(answers are in the back of the book)