

Stat 342 Example 10

Let's consider arguing directly that the pdf of

$$V = -\ln U$$

for $U \sim U(0,1)$ is the $\text{Exp}(1)$ pdf.

First, the pdf of U is $f_U(u) = \mathbb{I}[0 < u < 1]$.

For $h(u) = -\ln(u)$ for $u \in (0,1)$

$$h^{-1}(v) = \exp(-v) \text{ for } v > 0$$

and $h'(u) = -\frac{1}{u}$ for $u \in (0,1)$.

Then for $v > 0$

$$\frac{1}{|h'(h^{-1}(v))|} f(h^{-1}(v)) = \frac{1}{\left|-\frac{1}{\exp(-v)}\right|} \mathbb{I}[0 < \exp(-v) < 1]$$

$$= \exp(-v)$$

the $\text{Exp}(1)$ pdf.