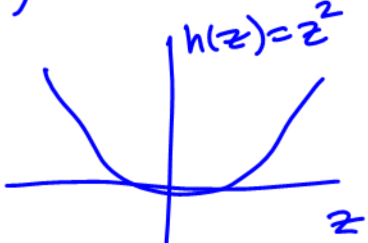
Stat 342 Example 11

Suppose that $Z \sim N(0,1)$. We consider the distribution of $W = Z^2$. (For $h(z) = z^2$ this is the dsn of $h(Z)$.) Note that $h(z)$ looks like



so that for $w > 0$ the cdf of W is

$$F(w) = P[W \le w] = P[Z^2 \le w] = P[-\sqrt{w} \le Z \le \sqrt{w}]$$
$$= \Phi(\sqrt{w}) - \Phi(-\sqrt{w}).$$

So the pdf for W is (for $w > 0$)

$$f(w) = \frac{d}{dw} F(w) = \phi(\sqrt{w}) \frac{1}{2\sqrt{w}} - \phi(-\sqrt{w})\left(-\frac{1}{2\sqrt{w}}\right)$$

$$= \phi(\sqrt{w}) \frac{1}{\sqrt{w}}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{w}} \exp -\frac{1}{2}(\sqrt{w} - 0)^2$$

$$= \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{w}} \exp -\frac{w}{2}$$

($F(w)$ is constant at 0 for $w < 0$ and so the density is 0 to the left of $w = 0$.) This is the χ^2_1 pdf, a special case of the Γ density important in elementary statistical inference.