Stat 342 Example 18

Suppose that $X \sim$ Poisson(λ) for a large value of λ ... say, e.g. $\lambda = 400$. How might one conclude that X is approximately normal?

It's easy to argue (using mgf's) that the sum of independent Poisson r.v.'s is Poisson. (The student should try this.)

So if, e.g. $Y_1, Y_2, \ldots, Y_{400}$ are iid Poisson (1) the sum $Y_1 + Y_2 + \cdots + Y_{400}$ is Poisson with $\lambda = 400$. But then the CLT says that $\bar{Y} = \frac{1}{400} \sum Y_i$ is approximately normal with mean $EY_1 = 1$ and std dev $\frac{\sqrt{\operatorname{Var} Y_1}}{\sqrt{400}} = \frac{1}{\sqrt{400}}$.

But that means that $\frac{X}{400}$ is approximately normal with mean 1 and std dev $\frac{1}{\sqrt{400}}$.

So, for example (ignoring corrections for discreteness)

$$P\left[380 \le X \le 420\right] = P\left[\frac{380}{400} \le \frac{X}{400} \le \frac{420}{400}\right]$$

$$= P\left[\underbrace{\frac{\frac{380}{400} - 1}{\frac{1}{\sqrt{400}}}}_{-1} \le \frac{\frac{X}{400} - 1}{\frac{1}{\sqrt{400}}} \le \underbrace{\frac{\frac{420}{400} - 1}{\frac{1}{\sqrt{400}}}}_{1}\right]$$

$$\approx \Phi(1) - \Phi(-1)$$