

Stat 342 Example 19

Suppose x_1, x_2, \dots, x_n are iid Exponential with mean θ (and therefore std dev θ as well).

The CLT says that

$$\frac{\bar{x} - \theta}{\frac{\theta}{\sqrt{n}}}$$

is approximately standard normal. So, for example

$$P\left[-1.96 \leq \frac{\bar{x} - \theta}{\frac{\theta}{\sqrt{n}}} \leq 1.96\right] \approx .95$$

this event is exactly
the event

$$\frac{\bar{x}}{1 + \frac{1.96}{\sqrt{n}}} \leq \theta \leq \frac{\bar{x}}{1 - \frac{1.96}{\sqrt{n}}}$$

That is, approximate 95% confidence limits for θ are

$$\frac{\bar{x}}{1 + \frac{1.96}{\sqrt{n}}} \quad \text{and} \quad \frac{\bar{x}}{1 - \frac{1.96}{\sqrt{n}}}$$