Stat 342 Example 20

Suppose that $x_1, x_2, \ldots, x_n$ are iid Bernoulli(p). Consider $\hat{p} = \bar{x} = \frac{\sum x_i}{n}$ and the r.v. $\exp(\hat{p})$. What is an approximate dsn for this variable?

The CLT says that

$$\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

is approximately std normal. ($\hat{p}$ is approximately normal with mean p and std dev $\sqrt{\frac{p(1-p)}{n}}$.) Then the delta method using $h(\cdot) = \exp(\cdot)$ implies that $\exp(\hat{p})$ is approximately normal with mean $\exp(p)$ and standard deviation $|h'(p)|\sqrt{\frac{p(1-p)}{n}}$ i.e. $\exp(p)\sqrt{\frac{p(1-p)}{n}}$.