

Stat 342 Example 2)

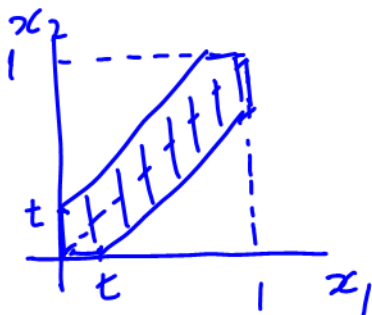
Suppose x_1 and x_2 are iid $U(0,1)$. How might one find the dsn of $|x_1 - x_2|$ and/or

$$P[|x_1 - x_2| < .1] ?$$

$$f(x_1, x_2) = \mathbb{I}[0 < x_1 < 1 \text{ and } 0 < x_2 < 1]$$

So for $t > 0$

$$P[|x_1 - x_2| < t] = \text{area of region pictured below}$$



$$= 1 - 2 \left(\text{area of } \Delta \right)$$

$$= 1 - 2 \left(\frac{1}{2} (1-t)^2 \right)$$

$$= 1 - (1-t)^2$$

$$\text{So } P[|x_1 - x_2| < .1] = 1 - .81 = .190$$

A simulation-based estimate of this can be made using R (or JMP for that matter). Here is some R code and the output it generates

```
x1<-c(runif(10000))
x2<-c(runif(10000))
abdiff<-abs(x1-x2)
hist(abdiff)
n<-length(abdiff)
plot(sort(abdiff),(1:n)/n,type="s",ylim=c(0,1))
length(abdiff[abdiff<.1])/10000
```

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> n<-length(abdiff)
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> length(abdiff[abdiff<.1])/10000
```

[1] 0.1931

approx probability
 $|x_1 - x_2| < .1$

Histogram of abdiff

