Stat 342 Example 24

Consider 2 pmfs for y given below

	y=1	2	3	4	5	6	7
θ=0	.2	.05	.05	.1	.2	.2	.2
θ=1	.05	.1	.1	.2	.1	.15	.3

Suppose I'm only told that $y \in \{2,3,4\}$ and not the exact value of y. Does this degrade my ability to do inference for θ? A way to think about this is as follows. Conditional on $y \in \{2,3,4\}$ the θ=0 dsn of y is

y	2	3	4
cond. prob.	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

$\frac{.05}{.05+.05+.1}$ $\frac{.1}{.05+.05+.1}$

while the θ=1 dsn of y is the same

y	2	3	4
cond. prob.	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

$\frac{.1}{.1+.1+.2}$ $\frac{.2}{.1+.1+.2}$

i.e. the conditional dsn doesn't depend upon θ. The "extra information" in the exact value of y beyond the information that $y \in \{2,3,4\}$ is of no value in inference about θ.

Note that in this example the likelihood ratios are:

y	1	2	3	4	5	6	7
$\Lambda_y(1,0)$	$\frac{1}{4}$	2	2	2	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$
$\Lambda_y(0,1)$	4	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	2	$\frac{4}{3}$	$\frac{2}{3}$

In light of the values of (say) $\Lambda_y(1,0)$

T_1 defined by

y	1	2	3	4	5	6	7
$T_1(y)$	6	10	10	11	35	2	1.7

and T_2 defined by

y	1	2	3	4	5	6	7
$T_2(y)$	6	10	10	10	16	12	-5

are both sufficient for θ. But T_3 defined by

y	1	2	3	4	5	6	7
$T_3(y)$	6	6	10	10	16	12	-5

is <u>not</u> sufficient for θ since $\Lambda_1(1,0) = \frac{1}{4}$ while $\Lambda_2(1,0) = 2$ while $T_3(1) = T_3(2) = 6$.

T_2 in this example is actually minimal sufficient (while T_1 is not) since $T_2(y) = T_2(y')$ exactly when $\Lambda_y(\theta, 0) = \Lambda_{y'}(\theta, 0)$ $\forall\ \theta$ (likelihood ratio functions with reference to $\theta = 0$ are the same).