Stat 342 Example 25

Suppose $x_1, x_2, \ldots, x_n$ are iid $N(\mu, 1)$

$$f(\underset{\sim}{x}|\mu) = \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}(x_i-\mu)^2\right) \right)$$

So $$\Lambda_{\underset{\sim}{x}}(\mu_1, \mu_2) = \frac{\exp\left(-\frac{1}{2}\sum(x_i-\mu_1)^2\right)}{\exp\left(-\frac{1}{2}\sum(x_i-\mu_2)^2\right)}$$

$$= \frac{\exp\left(\mu_1 \sum x_i - \frac{n}{2}\mu_1^2\right)}{\exp\left(\mu_2 \sum x_i - \frac{n}{2}\mu_2^2\right)}$$

Note that for $T(\underset{\sim}{x}) = \sum x_i$, all $\underset{\sim}{x}$ with $T(\underset{\sim}{x}) = t$ have the same likelihood ratio function of μ_1 and μ_2

$$\frac{\exp\left(\mu_1 t - \frac{n}{2}\mu_1^2\right)}{\exp\left(\mu_2 t - \frac{n}{2}\mu_2^2\right)}$$

so $T(\underset{\sim}{x}) = \sum x_i$ is sufficient for μ.