Stat 342 Example 26

Suppose that $x_1, x_2, \ldots, x_n$ are iid Bernoulli(p).
Then

$$f(\underset{\sim}{x} \mid p) = \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$$

$$= p^{\Sigma x_i} (1-p)^{\Sigma(1-x_i)}$$

$$= p^{\Sigma x_i} (1-p)^{n-\Sigma x_i}$$

So

$$\Lambda_{\underset{\sim}{x}}(p_1, p_2) = \frac{p_1^{\Sigma x_i} (1-p_1)^{n-\Sigma x_i}}{p_2^{\Sigma x_i} (1-p_2)^{n-\Sigma x_i}}$$

$$= \left(\frac{p_1}{p_2}\right)^{\Sigma x_i} \left(\frac{1-p_1}{1-p_2}\right)^{n-\Sigma x_i}$$

For $T(\underset{\sim}{x}) = \Sigma x_i$, for all $\underset{\sim}{x}$ with $T(\underset{\sim}{x}) = t$ we get the same likelihood ratio function of (p_1, p_2), namely

$$\left(\frac{p_1}{p_2}\right)^{t} \left(\frac{1-p_1}{1-p_2}\right)^{n-t}$$

So $T(\underset{\sim}{x}) = \Sigma x_i$ is sufficient for t.