Stat 342 Example 27

Consider the statistical model with $\theta \in \{0,1,2\}$ and pmf's for y given below

y	1	2	3	4	5	6	7
$\theta=2$	.05	.05	.2	.4	.2	.05	.05
$\theta=1$	.05	.1	.1	.2	.1	.15	.3
$\theta=0$	.2	.05	.05	.1	.2	.2	.2

The two likelihood ratio functions relative to $\theta=0$ are below

y	1	2	3	4	5	6	7
$\Lambda_y(1,0)$	$\frac{1}{4}$	2	2	2	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$
$\Lambda_y(2,0)$	$\frac{1}{4}$	1	4	4	1	$\frac{1}{4}$	$\frac{1}{4}$

The bivariate statistic $T(y) = (\Lambda_y(1,0), \Lambda_y(2,0))$ is minimal sufficient. Its values are (obviously) $(\frac{1}{4},\frac{1}{4})$, $(2,1)$, $(2,4)$, $(2,4)$, $(\frac{1}{2},1)$, $(\frac{3}{4},\frac{1}{4})$, $(\frac{3}{2},\frac{1}{4})$

(obviously the same)

So any mapping of $\{1,2,3,4,5,6,7\}$ onto 6 different values such that $y=3$ and $y=4$ get mapped to the same value will produce a minimal sufficient statistic.