

Stat 342 Example 28

Suppose $x \sim N(\mu, 1)$.

$$f(x|\mu) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-\mu)^2\right)$$

$$\ln f(x|\mu) = -\frac{1}{2} \ln(2\pi) - \frac{1}{2}(x-\mu)^2$$

$$\frac{\partial}{\partial \mu} \ln f(x|\mu) = x - \mu$$

$$\text{So } I_x(\mu) = \text{Var}_\mu(x - \mu) = \text{Var}_\mu(x) = 1$$

$$\text{Or, } \frac{\partial^2}{\partial \mu^2} \ln f(x|\mu) = -1 \quad \text{and}$$

$$I_x(\mu) = E\left(-\frac{\partial^2}{\partial \mu^2} \ln f(x|\mu)\right) = 1$$