

Stat 342 Example 29

Suppose $x \sim \text{Poisson}(\lambda)$

Then

$$\ln f(x|\lambda) = -\lambda + x \ln \lambda - \ln x!$$

$$\frac{\partial}{\partial \lambda} (\quad) = -1 + \frac{x}{\lambda}, \quad \text{so}$$

$$\begin{aligned} I_x(\lambda) &= \text{Var}_\lambda \left(-1 + \frac{x}{\lambda} \right) = \frac{1}{\lambda^2} \text{Var}_\lambda x \\ &= \frac{1}{\lambda^2} (\lambda) = \frac{1}{\lambda} \end{aligned}$$

Or

$$\frac{\partial^2}{\partial \lambda^2} (\ln f(x|\lambda)) = -\frac{x}{\lambda^2} \quad \text{and}$$

$$\begin{aligned} I_x(\lambda) &= E_\lambda \left(-\frac{\partial^2}{\partial \lambda^2} \ln f(x|\lambda) \right) = +\frac{1}{\lambda^2} E_\lambda x \\ &= \frac{\lambda}{\lambda^2} \\ &= \frac{1}{\lambda} \end{aligned}$$