Stat 342 Example 3

Suppose the entries of $\underset{\sim}{x} = (x_1, x_2, \dots, x_n)$ are iid Poisson λ and the inference problem concerns λ. The joint pmf of $\underset{\sim}{x}$ is

$$f(\underset{\sim}{x}|\lambda) = \prod_{i=1}^{n} \left(\frac{e^{-\lambda} \lambda^{x_i}}{x_i!} \right)$$

Then treated as a function of λ for data $\underset{\sim}{x}$ plugged in, a log-likelihood function is

$$\ln f(\underset{\sim}{x}|\lambda) = -n\lambda + \left(\sum_{i=1}^{n} x_i\right) \ln \lambda - \sum_{i=1}^{n} \ln(x_i!)$$

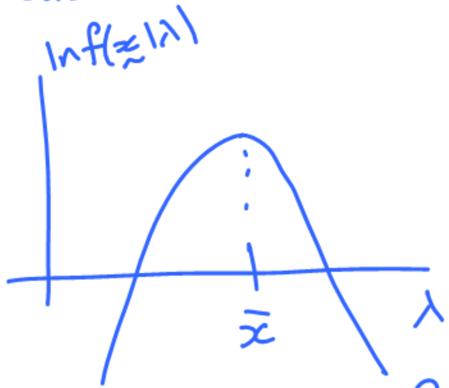
Then

$$\frac{d}{d\lambda} \ln f(\underset{\sim}{x}|\lambda) = -n + \frac{1}{\lambda}\left(\sum x_i\right)$$

For $\sum x_i = 0$ this is negative and so the smallest possible value of λ, namely $\lambda = 0$ maximizes the likelihood.
For other $\sum x_i$ one may set

$$\frac{d}{d\lambda} \ln f(\underset{\sim}{x}|\lambda) = 0$$

and solve for λ, obtaining $\lambda = \bar{x}$. A cartoon for the $\sum x_i > 0$ case is below.



Thus in general the likelihood is maximized at $\bar{x}$ and

$$\hat{\lambda}^{MLE}(\underset{\sim}{x}) = \bar{x}$$