

## Stat 342 Example 30

Suppose  $x$  and  $y$  are independent, both Poisson( $\lambda$ ). What is the FI in the pair  $(x, y)$  about  $\lambda$ ?

$$f(x, y | \lambda) = \frac{\exp(-\lambda) \lambda^x}{x!} \cdot \frac{\exp(-\lambda) \lambda^y}{y!}$$

so

$$\ln f(x, y | \lambda) = -\lambda + x \ln \lambda - \ln x! \\ -\lambda + y \ln \lambda - \ln y!$$

and then

$$\frac{\partial}{\partial \lambda} \ln f(x, y | \lambda) = -2 + (x+y) \frac{1}{\lambda}$$

$$\text{and } I_{(x, y)}(\lambda) = \text{Var}_{\lambda} \left( -2 + \frac{1}{\lambda} (x+y) \right)$$

$$= \frac{1}{\lambda^2} \text{Var}_{\lambda} (x+y)$$

independence  $\Rightarrow \oplus$

$$\frac{1}{\lambda^2} \left( \text{Var}_{\lambda} (x) + \text{Var}_{\lambda} (y) \right)$$

$$= \frac{2\lambda}{\lambda^2} = \frac{2}{\lambda}$$