Stat 342 Example 32

Suppose $\underset{\sim}{x} = (x_1, x_2, \ldots, x_n)$ has iid $N(\mu, 1)$ components x_i. The statistic $T(\underset{\sim}{x}) = \bar{x}$ has

$$E_\mu T(\underset{\sim}{x}) = E_\mu \bar{x} = \mu$$

and is thus unbiased for μ. The Cramér-Rao lower bound on the variance of an unbiased estimator of μ is

$$\mathrm{Var}_\mu \hat{\mu}(\underset{\sim}{x}) \geq \frac{1}{I_{\underset{\sim}{x}}(\mu)} = \frac{1}{n(1)} = \frac{1}{n}$$

But $\bar{x}$ has $\mathrm{Var}_\mu \bar{x} = \frac{1}{n} \mathrm{Var}\, x_1 = \frac{1}{n}$ and thus "achieves the C-R lower bound" (for every μ). So among all unbiased estimators of μ, $\bar{x}$ is best (it has "<u>u</u>niformly <u>m</u>inimum <u>v</u>ariance").

(in μ) ↑