Stat 342 Example 36

Suppose the random pair (x,y) has pmf $f(x,y|\theta)$ for $\theta = 0$ or 1 given below

$y \backslash x$	1	2	3
3	.15	.1	.05
2	.1	.2	.05
1	.1	.05	.2

$f(x,y|0)$

$y \backslash x$	1	2	3
3	.2	.15	.05
2	.2	.05	.1
1	.1	.1	.05

$f(x,y|1)$

Then the likelihood ratio statistic $\Lambda_{(x,y)}(1,0)$ is given in the table below

$y \backslash x$	1	2	3
3	$4/3$	$3/2$	1
2	2	$1/4$	2
1	1	2	$1/4$

Then, if we order the (x,y) points by the value of their $\Lambda_{(x,y)}(1,0)$, we get the following:

Λ	$1/4$	1	$4/3$	$3/2$	2
(x,y)	(3,1) (2,2)	(3,3) (1,1)	(1,3)	(2,3)	(1,2) (2,3) (3,2)

I can "cut" the value of $\Lambda_{(x,y)}(1,0)$ at any number, reject $H_0: \theta = 0$ to the "right" (on Λ) and have a most powerful test of its size.

For example,

$$a(x,y) = I\left[\Lambda_{(x,y)}(1,0) \geq 1.4\right]$$

$$= \begin{cases} 1 & \text{if } (x,y) = (2,3) \text{ or } (1,2) \text{ or } (2,3) \text{ or } (3,2) \\ 0 & \text{otherwise} \end{cases}$$

has size

$$R(0) = P_0\left[a(x,y) = 1\right]$$

$$= .1 + .05 + .05 + .1 = .3 = \alpha$$

$$R(1) = P_1\left[a(x,y) = 0\right]$$

$$= .05 + .05 + .05 + .1 + .2 = .45 = \beta$$

so the "power" of the test is

$$1 - \beta = 1 - .45 = .55$$