Stat 342 Example 37

Suppose $x \sim N(\mu, 1)$ and I wish to test $H_0: \mu = 0$ vs $H_a: \mu = 2$. For $f(x|\mu)$ the $N(\mu,1)$ density

$$\ln \frac{f(x|2)}{f(x|0)} = \ln \exp -\tfrac{1}{2}(x-2)^2 - \ln \exp -\tfrac{1}{2}x^2$$

$$= x - 2$$

so any good test of these simple hypotheses will reject H_0 for large $x-2$ i.e. large x.

Consider, for example a Bayes optimal test for a prior distribution that sets $g(2) = .8$ and $g(0) = .2$. A test with minimum Bayes risk has

$$a(x) = I\left[\frac{f(x|2)}{f(x|0)} > \frac{.2}{.8}\right]$$

$$= I\left[\exp(x-2) > \tfrac{1}{4}\right]$$

$$= I\left[x > 2 + \ln(.25)\right]$$

This test has size

$$P_0\left[x > 2 + \ln(.25)\right] = 1 - \Phi\left(2 + \ln(.25)\right)$$

and power

$$1 - \beta = 1 - P_2\left[x > 2 + \ln(.25)\right]$$

$$= 1 - \Phi\left(\ln(.25)\right)$$