Stat 342 Example 38

Suppose $x \sim N(\mu, 1)$ and consider testing $H_0: \mu = 0$ vs $H_a: \mu \neq 0$. Here the generalized log likelihood ratio is

$$\lambda(x) = \ln \frac{f(x \mid \hat{\mu}^{MLE})}{f(x \mid 0)} \qquad (\hat{\mu}^{MLE} = x)$$

$$= \ln \frac{\exp -\frac{1}{2}(x-x)^2}{\exp -\frac{1}{2}(x-0)^2}$$

$$= \frac{1}{2}x^2$$

So likelihood ratio tests here will be of form

$$a(x) = I\left[\tfrac{1}{2}x^2 > \#\right] \quad \text{for some } \# > 0$$

Notice here that under $\mu = 0$

$$P\left[2\lambda(x) > c\right] = P\left[2\left(\tfrac{1}{2}\right)x^2 > c\right]$$

$$= P\left[x^2 > c\right]$$

$$= \chi^2_1 \text{ probability to the right of } c$$

So that we get the "large n" χ^2_1 behavior of the test statistic already when $n = 1$!

Rejecting $H_0: \mu = 0$ when $x^2 > 3.841$ gives <u>exactly</u> $\alpha = .05$ here.