Stat 342 Example 39

Suppose $\underset{\sim}{x} = (y_1, y_2, \ldots, y_n)$ for the y_i iid $N(0, v)$ and consider testing $H_0: v = 1$ vs $H_a: v \neq 1$

Here

$$\lambda_n(\underset{\sim}{x}) = \ln f(\underset{\sim}{x} \mid \hat{v}_n^{MLE}) - \ln f(\underset{\sim}{x} \mid 1)$$

$$= -\frac{n}{2}\ln 2\pi - \frac{n}{2}\ln \hat{v}_n^{MLE} - \frac{\sum y_i^2}{2\hat{v}_n^{MLE}}$$

$$-\frac{n}{2}\ln(2\pi) - \frac{1}{2}\sum y_i^2$$

$$= -\frac{n}{2}\ln \hat{v}_n^{MLE} + \frac{n}{2}\left(\hat{v}_n^{MLE} - 1\right)$$

Consider the function $g(v) = -\frac{n}{2}\ln v + \frac{n}{2}(v-1)$

$g'(v) = \frac{n}{2}\left(-\frac{1}{v}+1\right)$ and $\therefore$ $g'(v) < 0$ if $v < 1$ and $g'(v) > 0$ if $v > 1$. Then since $g(1) = 0$ there are unique $d^L < 1$ and $d^U > 1$ that solve

$$2g(v) = \begin{array}{c}\text{upper 5\% pt}\\ \text{of } \chi^2_1 \text{ dsn}\end{array} = 3.841$$

One can solve for these numerically and end with a likelihood ratio test of approximate size .05 of the form

$$a(\underset{\sim}{x}) = I\left[\hat{v}_n^{MLE} < d^L \text{ or } \hat{v}_n^{MLE} > d^U\right]$$

(If you are having trouble following this, make up a few simulated data values and actually find d^L and d^U!)