Stat 342 Example 4

Again suppose that components of $\underset{\sim}{x} = (x_1, \dots, x_n)$ are iid Poisson λ. Consider estimators of λ

$$\hat{\lambda}_{c,d}(\underset{\sim}{x}) = c + d\bar{x}$$

and their SEL risk functions.

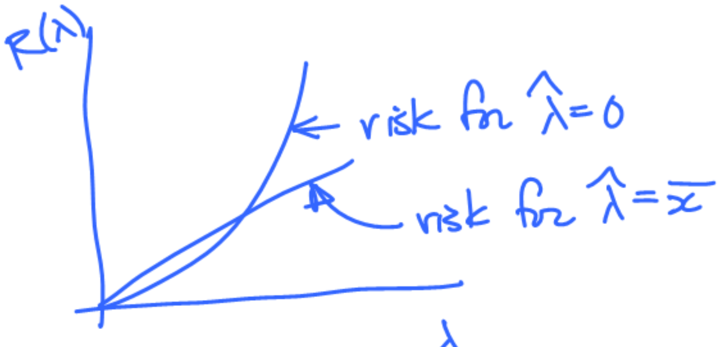
$$\underbrace{R(\lambda)}_{\text{MSE of estimation}} = \mathrm{Var}_\lambda\left(c+d\bar{x}\right) + \underbrace{\left(E_\lambda(c+d\bar{x}) - \lambda\right)^2}_{\text{squared bias of estimation}}$$

$$= d^2\,\mathrm{Var}_\lambda\,\bar{x} + \left(\left(c+d\lambda\right) - \lambda\right)^2$$

$$= d^2\left(\tfrac{1}{n}\lambda\right) + \left(\left(d-1\right)\lambda + c\right)^2$$

$$= \lambda^2(1-d) + \lambda\left(\frac{d^2}{n} - 2c(1-d)\right) + c^2$$

These are quadratic function of λ. Depending upon n and λ different values of c and d will be better. For example, consider the 2 cases of $c=0$ and $d=0$ (so $\hat{\lambda} \equiv 0$) and $c=1$ and $d=1$ (so $\hat{\lambda} = \bar{x}$). These have risk functions $R(\lambda) = \lambda^2$ and $R(\lambda) = \frac{\lambda}{n}$ respectively.



For example, the first is preferable if $\lambda^2 < \frac{\lambda}{n}$ i.e. if $\lambda < \frac{1}{n}$.