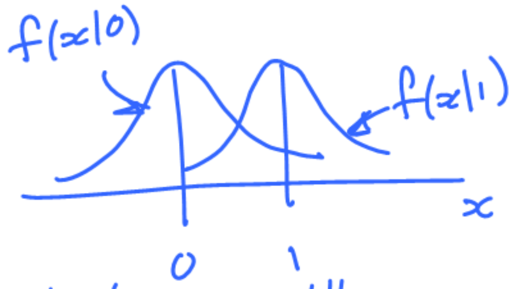
# Stat 342 Examples

Consider the "simple versus simple testing" problem for $\theta \in \{0,1\}$ with $f(x|\theta)$ the $N(\theta,1)$ density under 0-1 loss $\left(L(\theta,a) = I[a \neq \theta]\right)$. This is pictured below.




For illustration we'll consider first the decision function

$$a(x) = I\left[x > \tfrac{1}{2}\right] = \begin{cases} 1 & \text{if } x > \tfrac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

and find its risk function. This means that we need 2 expected losses, first for $\theta = 0$ and then for $\theta = 1$.

$$R(0) = E_0 I\left[a(x) \neq 0\right] = E_0 I\left[x > \tfrac{1}{2}\right] = P_0\left[x > \tfrac{1}{2}\right]$$
$$= 1 - \Phi\left(\tfrac{1}{2}\right)$$
$$R(1) = E_1 I\left[a(x) \neq 1\right] = P_1\left[x < \tfrac{1}{2}\right] = \Phi\left(-\tfrac{1}{2}\right)$$

the same

and this decision function has a constant risk function.

A more general decision function is

$$a_c(x) = I\left[x > c\right] = \begin{cases} 1 & \text{if } x > c \\ 0 & \text{otherwise} \end{cases}$$

This has a non-constant risk function since

$$R(0) = P_0\left[x > c\right] = 1 - \Phi(c)$$

and $$R(1) = P_1\left[x < c\right] = \Phi(c-1).$$