Stat 342 Example 7

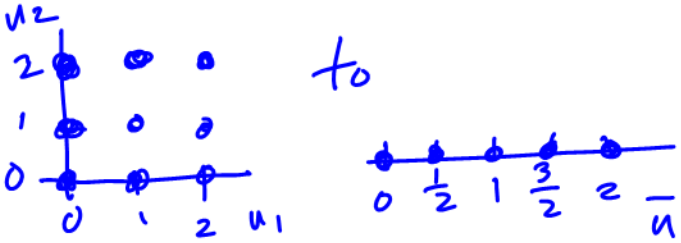
Suppose U_1 and U_2 are iid discrete r.v.'s with pmf

u	0	1	2
$f(u)$	.3	.4	.3

What is the distribution of $\bar{U} = \frac{1}{2}(U_1 + U_2)$? (This question is "what are possible values of $\bar{U}$ and the corresponding probabilities?")

(U_1, U_2) are jointly discretely distributed with pmf as

u_2 \ u_1	0	1	2	
2	.09	.12	.09	.2
1	.12	.16	.12	.4
0	.09	.12	.09	.3
	.3	.4	.3	

$h(u_1, u_2) = \frac{1}{2}(u_1 + u_2)$ maps  to

and we're looking for the dsn of $h(U_1, U_2)$. We get this by for each possible $\bar{u}$ adding probabilities of (u_1, u_2) pairs with average of u_1 and u_2 equal to that value. That is

$$P(\bar{U} = 0) = P[U_1 = 0 \text{ and } U_2 = 0] = .09$$

$$P(\bar{U} = 1) = P[U_1 = 0 \text{ and } U_2 = 1] + P[U_1 = 1 \text{ and } U_2 = 0] = .24$$

and so on, producing the pmf

$\bar{u}$	$f(\bar{u})$
0	.09
$\frac{1}{2}$	.12 + .12 = .24
1	.09 + .16 + .09 = .34
$\frac{3}{2}$	.12 + .12 = .24
2	.09