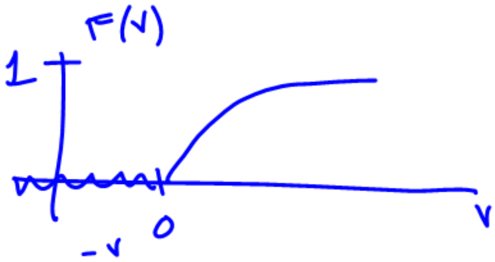
Stat 342 Example 9

Consider the "inverse probability integral transform" for converting a uniform (0,1) r.v. to an exponential r.v. The Exp(1) cdf is

$$F(v) = \begin{cases} 0 & \text{if } v<0 \\ 1-e^{-v} & \text{if } v>0 \end{cases}$$



So for $u \in (0,1)$, $F(v) = u \Rightarrow$

$$1-e^{-v} = u$$
$$1-u = e^{-v}$$
$$\ln(1-u) = -v$$
$$v = -\ln(1-u)$$

So by the result stated in class (Proposition 1 in the notes) for $U \sim U(0,1)$

$$V = -\ln(1-U) \sim Exp(1)$$

But if $U \sim U(0,1)$, so is $U^* = (1-U) \sim Exp(1)$.
That is, we've argued that $-\ln(U^*) \sim Exp(1)$ as desired.