Stat 342 Exam 1
Fall 2014

I have neither given nor received unauthorized assistance on this exam.

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Name Signed **Date**

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Name Printed

There are 10 questions on the following 5 pages. Do as many of them as you can in the available time. I will score each question out of 10 points AND TOTAL YOUR BEST 7 SCORES. (That is, this is a 70 point exam.)

1. Random variables x and y have a joint probability mass function specified in the table below.

	x = 1	2	3	4
y = 2	.08	.10	.12	.10
1	.06	.04	.11	.09
0	.06	.06	.07	.11

10 pts

a) Find the SEL optimal predictor of y based on x, say $\hat{y}^{\text{opt}}_{\text{SEL}}(x)$. (Specify the 4 values $\hat{y}^{\text{opt}}_{\text{SEL}}(1), \hat{y}^{\text{opt}}_{\text{SEL}}(2), \hat{y}^{\text{opt}}_{\text{SEL}}(3),$ and $\hat{y}^{\text{opt}}_{\text{SEL}}(4)$. There is no need to do the arithmetic.)

10 pts

b) Find the 0-1 loss predictor of y based on x, say $\hat{y}^{\text{opt}}_{\text{0-1}}(x)$. (Specify the 4 values $\hat{y}^{\text{opt}}_{\text{0-1}}(1), \hat{y}^{\text{opt}}_{\text{0-1}}(2), \hat{y}^{\text{opt}}_{\text{0-1}}(3),$ and $\hat{y}^{\text{opt}}_{\text{0-1}}(4)$.)

10 pts **c)** Suppose that one begins a Gibbs SSS algorithm at $(x^0, y^0) = (1,1)$. Give the distribution one uses to generate x^1. Then, supposing that $x^1 = 2$, give the distribution that one uses in order to generate y^1.

10 pts **d)** Find the distribution of the sum $t = x + y$. (List possible values and corresponding probabilities in a table below.)

10 pts **2.** In the planning of a large engineering project, the number of days required to complete one step in the project is modeled as a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 100 \\ \dfrac{(x-100)^3}{(100)^3} & \text{if } 100 \le x \le 200 \\ 1 & \text{if } x > 200 \end{cases}$$

a) A simulation is going to be used to study the feasibility of the project and variables with this distribution function are needed. For $U \sim \mathrm{U}(0,1)$ give a function $h(u)$ for which $h(U)$ has the target distribution (has cdf $F(x)$).

10 pts **b)** Approximate the probability that the sample mean of $n = 16$ simulated values from the distribution specified by F exceeds 180 days. (You may use without proof the fact that the distribution has mean 175 and standard deviation 19.4.)

10 pts **3.** The exponential distribution with mean μ has standard deviation μ. If $x_1, x_2, \ldots$ are iid according to this distribution, what is an approximate distribution for

$$\ln\left(\bar{x}_n\right)$$

(the natural logarithm of the sample mean of the first n of these variables)?

10 pts **4.** If $x_1, x_2, \ldots, x_n$ are iid $\mathrm{N}\left(\mu, \sigma^2\right)$ then we know that $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$. Let # stand for the lower 2.5% point of the χ^2_{n-1} distribution and ## stand for the upper 2.5% point of the χ^2_{n-1} distribution. Use these elements and produce 95% confidence limits for σ. That is, give formulas for random variables L and U so that in this context

$$P\left[L < \sigma < U\right] = .95$$

for all possible $\left(\mu, \sigma^2\right)$ pairs. (Explain/make clear why your limits will work.)

10 pts **5.** The Poisson(λ) pmf is $f(x|\lambda)=\lambda^x \exp(-\lambda)/x!$ for non-negative integer x. The $\Gamma(\alpha,\beta)$ pdf is $g(\theta)=\theta^{\alpha-1}\exp(-\theta/\beta)/\beta^\alpha\Gamma(\alpha)$ for $\theta>0$. (You may use this information in the following.)

In a Bayes model, $x_1,x_2,\ldots,x_n$ are iid Poisson(λ) and *a priori* $\lambda \sim \text{Exp}(1)$. Identify the posterior distribution.

10 pts **6.** A single random observation x has a distribution with pdf $f(x|\theta)=x^\theta(\theta+1)I[0<x<1]$. Find the maximum likelihood estimator of θ in this statistical model and write out (but do NOT attempt to evaluate/simplify) a completely specified definite integral that gives the squared error loss risk function of this estimator, $R(\theta)$.