Consider a statistical model where a single discrete observation X has probability mass function $f(x|\theta)$ indicated in the table below.

		x = 0	1	2	3	4	5
θ	5	.1	0	.2	.4	.2	.1
	4	.1	.3	.2	.2	.1	.1
	3	.1	0	.2	.4	.2	.1
	2	.2	.1	.4	0	0	.3
	1	.2	.1	.4	.2	.1	0

a) Identify a minimal sufficient statistic in this model, $T(X)$. Argue carefully that it really is minimal sufficient.

b) Suppose that a Bayesian uses a prior distribution for θ that is uniform on $\Theta = \{1,2,3,4,5\}$. Show that this person's posterior distribution for θ depends on X only through $T(X)$ (your minimal sufficient statistic from **a)**). (Hint: What are the 6 possible posterior distributions of θ, $g(\theta|x)$, for $x = 0,1,2,3,4,5$?)

c) Identify a most powerful test of size $\alpha = .05$ for testing $\mathrm{H}_0 : \theta = 1$ versus $\mathrm{H}_\mathrm{a} : \theta = 5$.

d) Identify a (0-1 loss) Bayes test of $\mathrm{H}_0 : \theta = 1$ or 2 versus $\mathrm{H}_\mathrm{a} : \theta = 3, 4$ or 5 if a prior uniform on $\Theta = \{1,2,3,4,5\}$ is used.

e) What is the size of your test from **d)**?

f) What is the likelihood ratio statistic for the hypotheses in **d)**? Is your test from **d)** a likelihood ratio test?

g) Describe how you would propose to test $\mathrm{H}_0 : \theta = 1$ versus $\mathrm{H}_\mathrm{a} : \theta = 5$ based on $X_1, X_2, \ldots, X_n$ that are iid with pmf $f(x|\theta)$. (Explicitly identify an appropriate test statistic. Do not try to work out either an exact or approximate cut-off value in explicit terms, but if you can see how derivation of one would go, indicate that in general terms.)