Stat 543 II

Suppose that $X \sim$ Bernoulli(p) and $Y \sim$ Normal$(\mu, 1)$ are independent and that one observes the random vector (X, Z) for $Z = XY$. (One may think of this situation as one of "noninformative censoring" where unless $X = 1$ one fails to observe Y.) The vector parameter for this model is (p, μ). The hypothesis H_0:$p\mu \geq 5$ is of interest.

(a) **Find** the mean and variance of Z.

(b) A possible test here is

$$\phi(x,z) = \begin{cases} 1 & \text{if } x = 1 \text{ and } z < 5 - 1.645 \\ 0 & \text{otherwise.} \end{cases}$$

Find the power function of this test, $\beta(p, \mu)$, **and** the size of this test for testing H_0:$p\mu \geq 5$.

(c) **Argue** very carefully that the test in (b) is a likelihood ratio test of H_0 versus H_a:not H_0. (Hint: You should consider (x, z) pairs of three types: $(0, 0)$, $(1, z)$ for $z \geq 5$ and $(1, z)$ for $z < 5$.)

(d) In contrast to the model indicated above, (in this part of the question only) consider the possibility that $Y \sim$ Normal$(\mu, 1)$, $X = I[Y > 0]$ and $Z = XY$. (This is a case where the censoring is informative. The censoring mechanism carries some information about μ.) **Find** the power function of the test from (b) under this model. (This will be a function of μ only. X here is still Bernoulli, but it is clearly dependent upon Y and"p" here is a function of μ.)

Now for large n, suppose that (X_i, Z_i) for $i = 1, ..., n$ are independent, each with distribution described at the top of the page.

(e) Keeping in mind the result of part (a), **give** consistent estimators of both p and $p\mu$.

(f) Based on the CLT, **propose** an approximately size $\alpha = .05$ test of H_0:$p\mu \geq 5$ versus H_a:$p\mu < 5$ that is a function of $(\overline{x}, \overline{z})$. (Hint: Consider the large sample distribution of $\overline{Z}$. Be sure your test statistic doesn't depend upon any unknown parameters.)

(g) Carefully **describe** how you could use simulation to approximate the power of your test from (f) when $n = 20$, $p = .5$ and $\mu = 13$.