

## Stat 543 II

Suppose that for a parameter  $\theta \in [-2, 2]$ ,  $X$  is a random variable with probability density

$$f(x|\theta) = \begin{cases} \theta x + \frac{1}{2}(2 - \theta) & \text{for } 0 < x < 1 \\ 0 & \text{otherwise .} \end{cases} \quad (*)$$

For parts a) and b), suppose further that *a priori*  $\theta$  is modeled as uniform on the finite set  $\{-1, 0, 1\}$ .

- Suppose that one observes  $X = .7$ . Find the posterior distribution of  $\theta$  given this outcome. (This will be a discrete distribution on the set  $\{-1, 0, 1\}$ .)
- Would a Bayes test reject  $H_0:\theta = 0$  in favor of  $H_a:\theta = -1$  or  $\theta = 1$ , upon observing  $X = .7$ ? Explain.

Drop the Bayesian assumption and now consider  $n$  independent observations from the distribution  $(*)$ ,  $X_1, X_2, \dots, X_n$ .

- Find the form of most powerful tests of the hypotheses  $H_0:\theta = 0$  vs  $H_a:\theta = 1$  based on  $n$  observations. (You don't need to do anything here beyond giving the form of an appropriate test statistic and saying for what kinds of values of this statistic one should reject  $H_0$ .)
- For the case of  $n = 2$  show how you would obtain a most powerful size  $\alpha = .05$  test of the hypotheses in c). (Figure 1 attached to this question is a contour plot of the joint density for  $(X_1, X_2)$  when  $\theta = 1$ . This plot may help you in this part. You don't need to actually evaluate integrals required to completely identify the correct rejection region, but set them up and say how they should be used.)

A particular sample of size  $n = 20$  from the distribution  $(*)$  produced the loglikelihood plotted in Figure 2.

- Suppose that it is of interest to test  $H_0:\theta \leq -1.2$  vs  $H_a:\theta > -1.2$ . What statistic would you use to do a likelihood ratio test of these hypotheses? What is the value of that statistic for the sample that led to Figure 2?
- Consider a likelihood ratio test of  $H_0:\theta = -1.2$  vs  $H_a:\theta \neq -1.2$ . Based on the large sample distribution of the likelihood ratio statistic in this problem, would the sample that led to Figure 2 cause rejection of  $H_0$  if  $\alpha = .05$ ? Explain carefully.

Figure 1

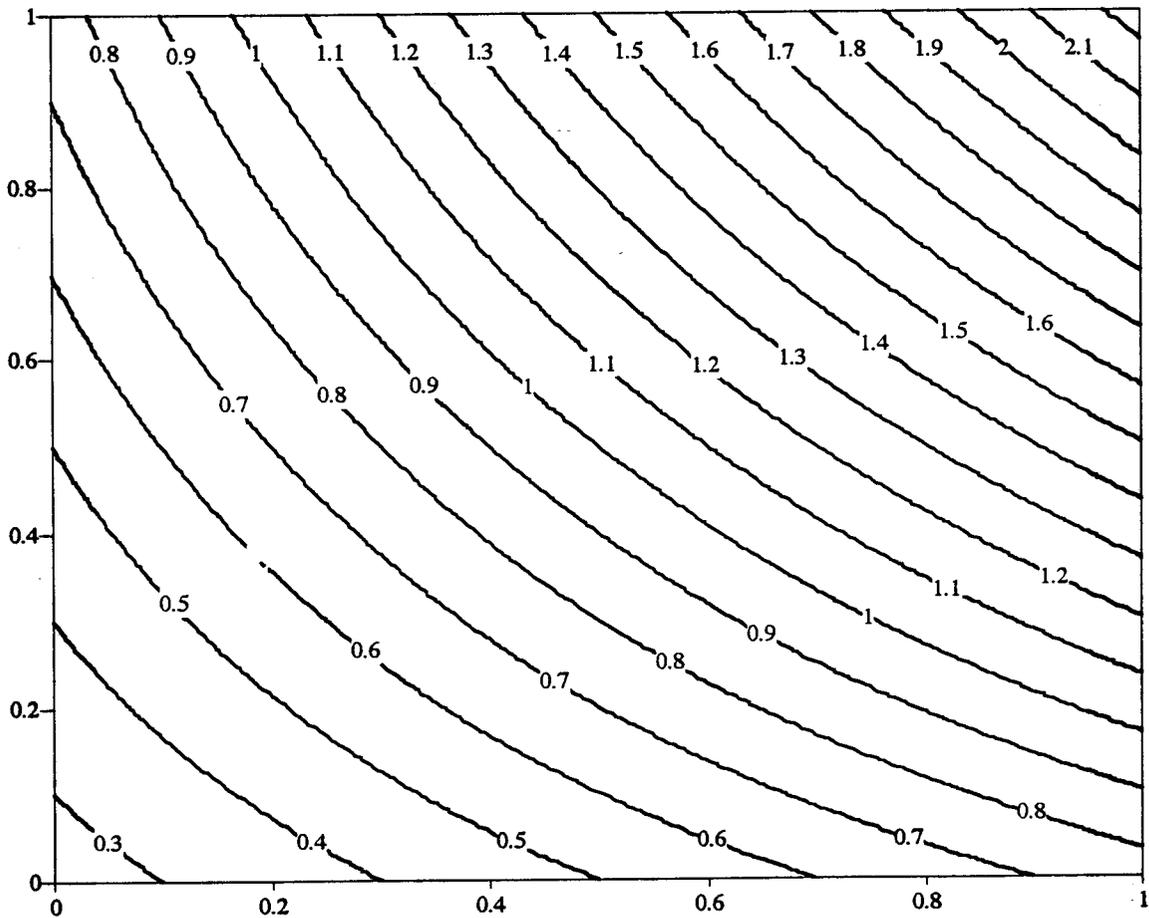


Figure 2

