

Stat 543 HW #8 (Due 4/15/16)
Monotone Likelihood Ratio, and UMP Tests, Likelihood Ratio Tests, Bayes and non-Bayes Intervals, Duality Between Tests and Confidence Procedures

1. B&D Problems 4.3.1,4.3.2, 4.3.5, 4.3.6, 4.3.9, 4.3.11

2. (Optional) Prove the following “filling-in” lemma:

Suppose that g_0 and g_1 are two different positive probability densities defined on an interval in \mathcal{R}^1 . If the ratio g_1/g_0 is nondecreasing in a real-valued function $T(x)$, then the family of densities $\{g_\alpha \mid \alpha \in [0, 1]\}$ for $g_\alpha = \alpha g_1 + (1 - \alpha)g_2$ has the MLR property in $T(x)$.

3. (Optional) Two possible definitions of "UMP size α " are:

Definition 1 A test ϕ of $H_0:\theta \in \Theta_0$ vs. $H_a:\theta \in \Theta_1$ is UMP of size α provided

- i) it is of size α , and
- ii) for any other test ϕ' of size α ,
 $\pi_\phi(\theta) \geq \pi_{\phi'}(\theta) \quad \forall \theta \in \Theta_1$

Definition 2 ... as in Definition 1, except in ii), let ϕ' be of size $\leq \alpha$

At first glance, it may seem that Definition 1 is weaker than Definition 2 (it might appear that ϕ could satisfy Definition 1 and fail to satisfy Definition 2). But, in fact, these two definitions are equivalent. Show the equivalence.

(Hint: If ϕ were to satisfy Definition 1 but not Definition 2, there would need to be a test ϕ' with $\alpha' = \sup_{\theta \in \Theta_0} \pi_{\phi'}(\theta) < \alpha$, such that for some

$\theta^* \in \Theta_1$, $\pi_{\phi'}(\theta^*) > \pi_\phi(\theta^*)$. Consider the test $\phi''(x) = \left(\frac{1-\alpha}{1-\alpha'}\right)\phi'(x) + \left(1 - \left(\frac{1-\alpha}{1-\alpha'}\right)\right)1$.)

4. Problems 4.9.1, 4.9.3, 4.9.9, 4.9.10 B&D

5. Problems 4.7.2, 4.4.1, 4.4.2, 4.8.1, 4.8.3 B&D

6. Problem 4.5.1 B&D

7. Optional (recommended but not required) Problems 4.4.7, 4.5.4, 4.7.1, 4.8.4 B&D